工程數學

核心能力: 學會求解工程問題中一階及高階常微分方程

- 正合微分方程、一階線性常微分方程
- 高階線性常微分方程
- 初始值問題、邊界值問題

1. $(x+\sin y)dy+(y+\cos x)dx=0$

2. $y'-y=e^{-x}$

3. $y''-3y'+2y=x+2$

4. $y''-3y'+2y=e^{3x},\quad y(0)=0, y'(0)=0$

核心能力: 學會求解工程問題中向量分析的概念

- 空間向量之向量微分梯度、散度及旋度的定義
- 線積分、面積分及體積分的物理意義

5. 若$\Phi=\sqrt{x^2+y^2},\quad \vec{V}=-x^2\,\vec{i}+2xy\,\vec{j}+(y-1)\,\vec{k}$, 試求$\mathrm{G}rad\,\Phi$, $Div\,\vec{V}$ 及$\mathrm{C}url\,\vec{V}$

6. 若 c 為圓心位於原點，半徑$r=a$之圓，力場向量$\vec{F}=y\vec{i}-x\vec{j}$，試求線積分$\oint_c \vec{F}\cdot d\vec{R}$。

核心能力: 學會求解工程問題中線性代數的概念

- 利用反矩陣法或克蘭默法則求解線性方程組
- 求解方陣之特徵值與特徵向量

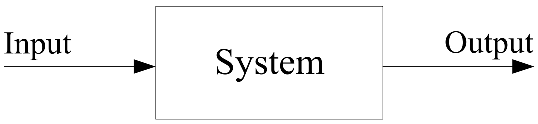
7. 若$\mathbf{A}=\begin{bmatrix}2 & 1 & -1\\ 1 & -3 & 1\\ 1 & 3 & -3\end{bmatrix}$為一方陣，$\mathbf{B}=\begin{bmatrix}5\\ 2\\ 0\end{bmatrix}$亦為一矩陣，試反矩陣法或克蘭默法則求解線性方程組$\mathbf{A}\,\mathbf{X}=\mathbf{B}$之解向量$\mathbf{X}=$?

8. 已知$\mathbf{A}=\begin{bmatrix}2 & 0 & 0\\ 2 & 1 & 1\\ 0 & 0 & 3\end{bmatrix}$，試求$\mathbf{A}$之特徵值及特徵向量。

車輛動態與控制

核心能力一　轉移函數之控制系統

1. 何謂轉移函數?



$$T(s)=\frac{Y(s)}{U(s)}\Rightarrow Y(s)=T(s)\cdot U(s)$$

2. 何謂頻率響應?

Δ frequency Response：系統輸入為正弦波←　討論的 Frequency 輸出

Consider a linear time–invariant stable system G(S).
If the input is $A\sin wt$, then the steady- state output is $B\sin(wt+\varphi)$ where

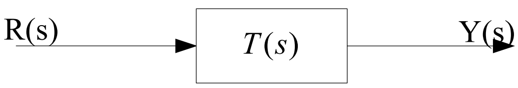
$$B=A\left|G(jw)\right|\text{ , }\varphi=\angle G(jw)=\tan^{-1}\left[\frac{G(jw)\text{的實部}}{G(jw)\text{的虛部}}\right]$$

Thus $G(jw)=\left|G(jw)\right|\angle G(jw)$

複數表示法　相量表示法

3. 何謂波德圖?

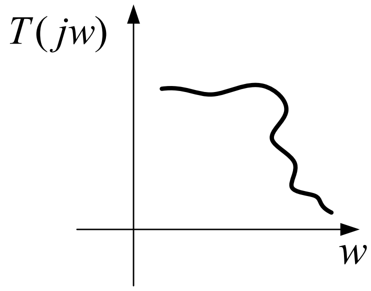
Δ Bode Plot　波德圖



$$T(jw)=\left|T(jw)\right|\angle T(jw)\equiv M\angle\phi$$

As different from polar plot, Bode plots are sketched in two parts, namely the magnitude part and the phase part.

(1) Magnitude of $\left|G(jw)\right|$ is drawn versus the frequency w



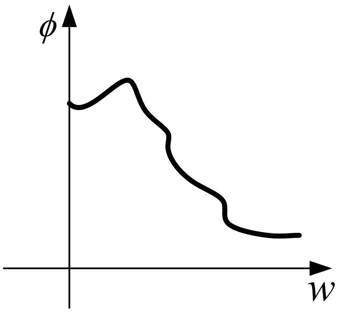
For convenience, the magnitude is plotted based on the logarithmic operator
(取 log 是為了方便相加)

$\because$ $|G_1(jw)G_2(jw)\cdots G_n(jw)| = |G_1(jw)||G_2(jw)|\cdots|G_n(jw)|$

if we take logarithmic operation , then $\log|G_1G_2\cdots G_n| = \log|G_1| + \log|G_2| + \cdots + \log|G_n|$

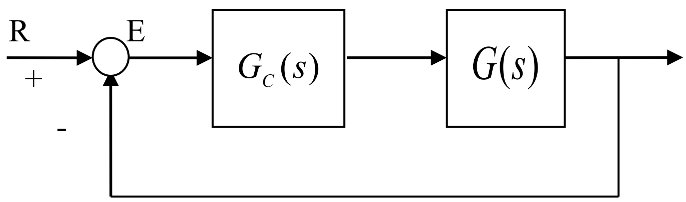
which can be obtained by adding all the individual $\log|G_i|$ together

(2) Phase of $T(jw) = T(jw)\angle T(jw)$, $\phi = \angle T(jw)$
If $G(jw) = G_1(jw)G_2(jw)\cdots G_n(jw)$
$\Rightarrow \angle G(jw) = \angle G_1(jw) + \angle G_2(jw) + \cdots + \angle G_n(jw)$
By adding all $\angle G_i(jw)$ together, $\angle G(jw)$ is then obtained.



4. 比例、微分、積分控制器之形式及特性為何?

控制器分類



(1) 比例控制(proportional control)

$G(s) = K_P$

特性：

i. 改善穩定度

ii. 改善穩態誤差
iii. 降低靈敏度
iv. 抑制雜訊

(2) 微分控制 (derivative control)

$$G(s) = K_D S$$

特性：
1. 改善穩定度
2. 提升響應速度
3. 增加雜寬
4. 放大雜訊 (x)

(3) 積分控制 (integral control)

$$G(s) = \frac{K_I}{S}$$

特性：
1. 增加系統 type 消除穩態誤差
2. 降低響應速度 (x)
3. 易造成系統不穩定 (x)

(4) 比例+微分 (P_D)

$$G(s) = K_P + K_D S$$

(5) 比例+積分 (P_I)

$$G(s) = K_P + \frac{K_I}{S}$$

(6) 比例+積分+微分 (P_{ID})

$$G(s) = K_P + K_D S + \frac{K_I}{S}$$

例題 1:

When the system shown in the Figure 1-(a) is subject to a unit step input the system output responds as shown in Figure 1－(b) Determine the values of T and K from the response curve.

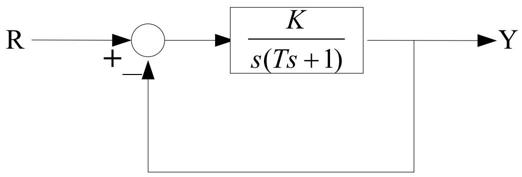
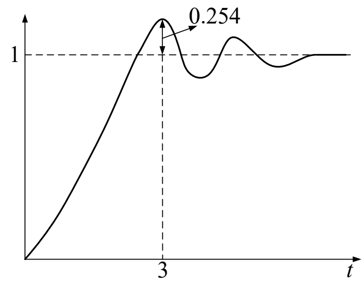
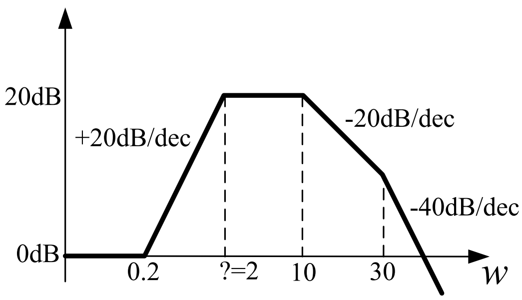


Figure 1-(a)



例題 2:

Estimate the transfer for the" Bode plot" for the magnitude shown in the figure. Assume it is minimum phase.



$$G(s)=\frac{K(s+2)}{(s+2)(s+10)(s+30)}$$

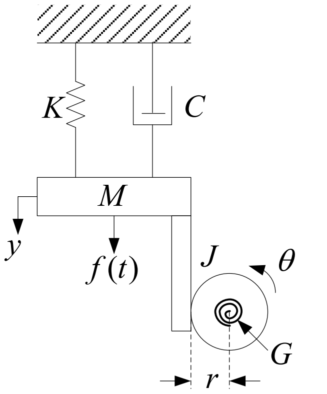
$G(0)=\frac{0.2K}{2\times10\times30}=1$ $\therefore$ K=3000

$\because 20\log 1 = 0\,\text{dB}$ → $|s|=1$

$$\therefore\ G(s)=\frac{3000(s+0.2)}{(s+2)(s+10)(s+30)}$$

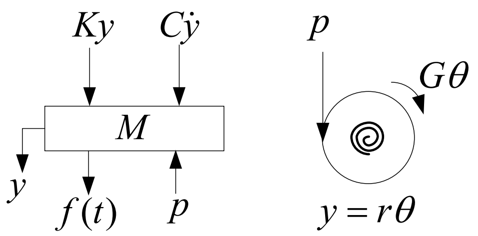
例題 3:

A mechanical system is shown in Figure. The motion of mass M is small and is constrained by a spring with constant k and a damper with coefficient c. A rigid bar is used to connect the mass and the disk. The motion of the disk is constrained by a torsional spring with spring constant G. The inertia of the disk is *J* and the friction of the disk is negligible. Derive the transfer function of this system in which *y* is the output and *f(t)* is the input.



Sol:

The free body diagrams are shown below



The dynamic equation

$f - p - ky - c\dot{y} = M\ddot{y}$ ..... (1)

$pr - G\theta = J\ddot{\theta}$ .... (2)

Because of $\theta = \frac{y}{r}$

(2)$\Rightarrow p = J\frac{\ddot{y}}{r^2} + G\frac{y}{r^2}$ into (1)

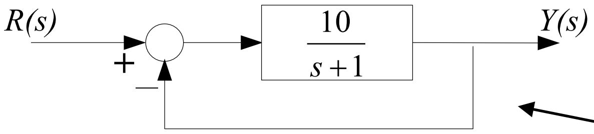
$\Rightarrow (M + \frac{J}{r^2})\ddot{y} + c\dot{y} + (K + \frac{G}{r^2})y = f$

Taking Laplace transforms and we can get the transfer function of the system

$$\frac{Y(s)}{F(s)} = \frac{1}{(M + \frac{J}{r^2})s^2 + cs + (K + \frac{G}{r^2})}$$

例題 4:

例：Consider a system shown in Figure. Suppose $r(t) = \sin(t + 60^\circ) - 2\cos(2t - 30^\circ)$ Find the steady- state response *y(t)*



重疊定理響應

Sol：

$$G(S)=\frac{Y(s)}{R(s)}=\frac{\frac{10}{s+1}}{1+\frac{10}{s+1}}=\frac{10}{s+11}$$

(1) $r(t)=\sin(t+60^\circ), w=1$

$$|G(j1)|=\left|\frac{10}{j1+11}\right|=\frac{10}{\sqrt{122}}$$

虛　實

$$\angle G(j1)=-\tan^{-1}\frac{1}{11}$$

$$\therefore y(t)=\frac{10}{\sqrt{122}}\sin(t+60^\circ-\tan^{-1}\frac{1}{11})$$

(2) $r(t)=-2\cos(2t-30^\circ), w=2$

$$|G(j2)|=\left|\frac{10}{j2+11}\right|=\frac{10}{\sqrt{125}}=\frac{2}{\sqrt{5}}$$

$$\angle G(j2)=-\tan^{-1}\frac{2}{11}$$

$$\therefore y(t)=-\frac{4}{\sqrt{5}}\cos(2t-30^\circ-\tan\frac{2}{11})$$

$\Rightarrow$ the total response

$$y(t)=\frac{10}{\sqrt{122}}\sin(t+60^\circ-\tan^{-1}\frac{1}{11})-\frac{4}{\sqrt{5}}\cos(2t-30^\circ-\tan^{-1}\frac{2}{11})$$

核心能力二 狀態空間之控制系統

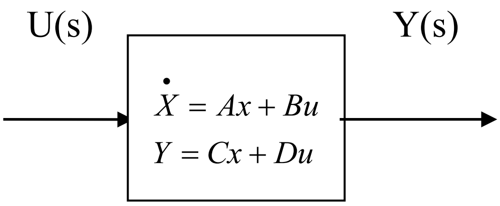
1. 系統之可控制性

狀態空間設計(State space design)

古典控制

$$G(s)=\frac{Y(s)}{U(s)}$$

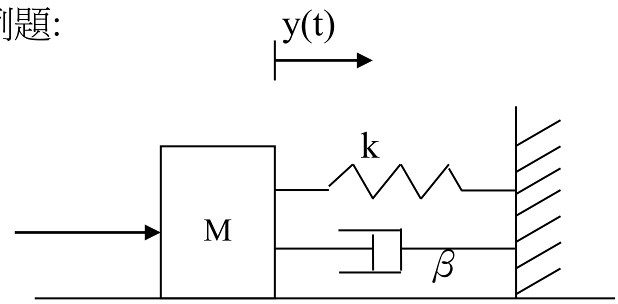
近代控制



$$\dot{X} = Ax + Bu$$
$$Y = Cx + Du$$

X:狀態變數 $x=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in R^n$ $\quad A\in R^{n\times n},\ B\in R^{n\times 1},\ C\in R^{1\times n},\ D\in R$

例題:



$$\mathrm{M}\ddot{y}(t)+B\dot{y}(t)+ky(t)=u$$

轉移函數法：

$$(MS^2+BS+k)Y(s)=U(s)$$

$$\therefore \frac{Y(s)}{U(s)}=\frac{1}{MS^2+BS+k}$$

狀態空間法：

令$y(t)=x_1(t)$ ， $\dot{y}(t)=x_2(t)$

(controllable canonical form)

$$\dot{x}_1=x_2$$

$$\dot{x}_2=\ddot{y}=-\frac{k}{M}y-\frac{B}{M}\dot{y}+\frac{1}{M}u$$

$$=-\frac{k}{M}x_1-\frac{B}{M}x_2+\frac{1}{M}u$$

$$\Rightarrow\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\frac{k}{M} & -\frac{B}{M}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ \frac{1}{M}\end{bmatrix}u$$

$$y=x_1=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$\therefore \dot{x}=Ax+Bu$$

$$y=Cx+Du$$

$$x=\begin{bmatrix}x_1\\ x_2\end{bmatrix}\qquad A=\begin{bmatrix}0 & 0\\ \frac{-k}{M} & -\frac{B}{M}\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 0\end{bmatrix}$$

$$D=0$$

狀態空間之控制器設計(Controller Design in State Space)

可控制性(Controllability)

若存在 $u=-kx$

$$\dot{x}=Ax+Bu \quad =-\begin{bmatrix}k_1 & k_2 & \ldots k_n\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix}$$

$$y=Cx$$

$$x=\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix}\in R^n$$

$$u,y\in R$$

$$A\in R^{n\times n}$$

$$C\in R^{1\times n}$$

$$B\in R^{n\times 1}$$

使得 $\dot{x}=(A-Bk)x$

其中$A-Bk$之特徵值得以任意決定，則稱此系統可控制，u為系統之狀態回授($state\ feedback$)控制器

條件：

控制性矩陣

$$Q_c = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$$

若 $Rank(Q_c) = n$ 則系統可控制或$\left| Q_c \right| \neq 0$亦可為判斷依據

2. 何謂狀態回授控制?

控制器設計

$$\dot{x} = Ax + Bu$$

$$u = -kx = -\begin{bmatrix} k_1 & k_2 & \dots & k_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\dot{x} = (A - Bk)x$$

$$= (A - \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \begin{bmatrix} k_1 & k_2 & \dots & k_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

1. 先決定控制閉迴路系統之理想極點或理想特徵值
2. $\lambda(A - Bk) =$ 理想值

   求出適當k值

例題 1: 一控制系統其狀態方程式如下:

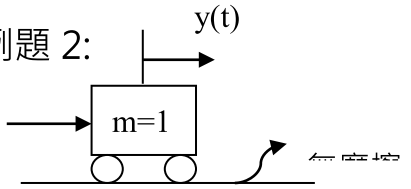
$$\dot{x}_1 = x_1 - 3x_2$$
$$\dot{x}_2 = 8x_1 + u$$

如果控制器設計為

$$u = -ax_1 - bx_2$$

其中 a、b 為常數。當此閉迴路控制系統為穩定時，求出 a、b 之範圍並以 a 為橫軸，b 為縱軸，畫出 a-b 之關係圖。

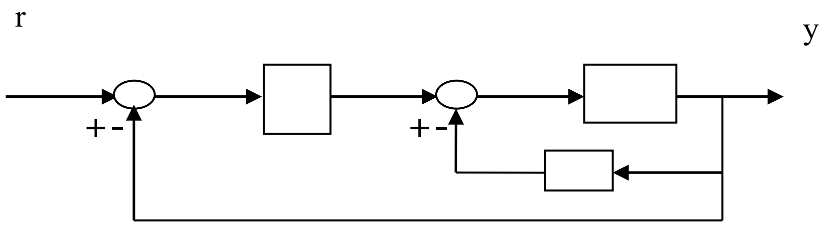
例題 2:



(1)定義狀態變數 $x_1 = y$, $x_2 = \dot{y}$ 以及輸出 y，列出狀態方程式。

(2)設計狀態回授控制器 $u = 2r - \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ 使得閉迴路特徵值為 $-1 \pm j$

(3)將此系統以下列方塊圖表示



Sol:

(1)

$$m\frac{d^2 y(t)}{dt^2} = u(t) \cdot m = 1$$

$$\ddot{y} = u, \quad \because x_1 = y,\ x_2 = \dot{y}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = u \quad \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = x_1 \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(2)

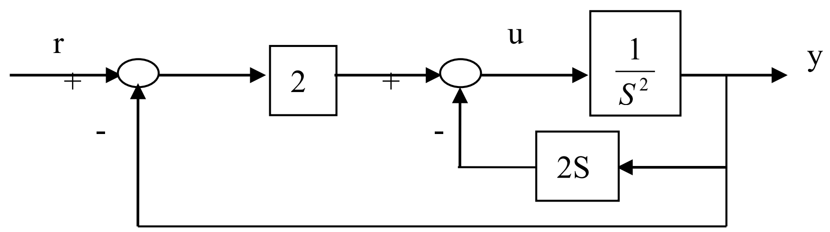
$u = 2r - \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ 代入

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\left\{\begin{bmatrix}0&1\\0&0\end{bmatrix}-\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}k_1&k_2\end{bmatrix}\right\}\begin{bmatrix}x_1\\x_2\end{bmatrix}+2r$$
$$=\begin{bmatrix}0&1\\-k_1&-k_2\end{bmatrix}\begin{bmatrix}x_1.\\x_2\end{bmatrix}+2r$$

$$\begin{vmatrix}\lambda&-1\\k_1&\lambda+k_2\end{vmatrix}=(\lambda+1+j)(\lambda+1-j$$
$$\lambda^2+k_2\lambda+k_1=\lambda^2+2\lambda+2$$
$$k_1=k_2=2$$

(3)

$$u=2r-2x_1-2x_2$$
$$=2(r-x_1)-2x_2$$



例題 3:

Give a system $G(s)=\dfrac{4s^2+25s+38}{s^3+9s^2+26s+24}$, if the state space representation of the system is

$$\dot{x}=\begin{bmatrix}-2&0&0\\0&a&0\\0&0&b\end{bmatrix}x+gu \qquad 求 a,b,g?$$
$$y=\begin{bmatrix}1&1&1\end{bmatrix}x$$

Sol:

$$G(s)=\frac{4s^2+25s+38}{s^3+9s^2+26s+24}$$
$$=\frac{A}{s+2}+\frac{B}{s-a}+\frac{C}{s-b}$$

$s^3+9s^2+26s+24=(s+2)(s+3)(s+4)$

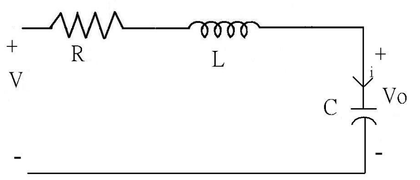
$a=-3$, $b=-4$

$A=2$ $B=1$ $C=1$

$$g=\begin{bmatrix}2\\1\\1\end{bmatrix}$$

例題 4:

例:



V 為輸入電壓，V_C 為輸出電壓
求狀態方程式及轉移函數

Sol：

電路方程式為

$$\begin{cases} L\dfrac{di}{dt}+Ri+V_C=V \\ C\dfrac{dVi}{dt}=i \end{cases}$$

轉移函數法(兩邊取拉式轉換)

$(LS+R)I(s)+V_C(s)=V(s)$

$CSV_C(s)=I(s)$

$[(LS+R)CS+1]V_C(s)=V(s)$

$$\frac{V_C(s)}{V(s)}=\frac{1}{CS(LS+R)+1}$$

狀態空間法

令 $V_C=\mathrm{x}_1$ $\mathrm{i}=\mathrm{x}_2$

$$\dot{x}_1 = \frac{1}{c} x_2$$

$$\dot{x}_2 = -\frac{1}{L} x_1 - \frac{R}{L} x_2 + \frac{1}{L} V$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{c} \\ \frac{-1}{L} & \frac{-R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} V$$

$$V_C = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

材料力學(Mechanics of Materials)

Text Book: 1. Statics and Mechanics of Materials (R. C. Hibbeler)

2. Mechanics of Materials (R. C. Hibbeler)

<u>核心能力 1: 材料機械性質</u>

a. 瞭解各種材料機械性質的定義

b. 瞭解如何由材料實驗得知相關機械性質，進而成為安全設計的參考值

1-1. Please define the meaning of: (a) homogeneous material, (b) isotropic material, (c) engineering stress, (d) engineering strain, (e) Poisson's ratio, (f) modulus of elasticity, (g) strength, (h) stiffness, (i) ductility, (j) toughness, (k) factor of safety, (l) fatigue, and (m) creep. (觀念-term definition)

1-2. Describe the events that occur when a specimen undergoes a tension test. Sketch a plausible engineering stress-strain curve and identify all significant regions and points between them. Assume that loading continues up to fracture. (觀念-stress-strain curve)

1-3. A 100-mm long rod has a diameter of 15mm. If an axial tensile load of 10kN is applied to it, determine the change in its diameter. E(elastic modulus)=70GPa, ν (Poisson's ratio)=0.35. (計算-tensile test)

1-4. The pin in Fig. 1 is made of a material having a failure shear stress of τ_{fail}=100MPa. Determine the minimum required diameter of the pin to the nearest mm. Apply a factor of safety F.S.=2.5 against shear failure. (計算-safety design)

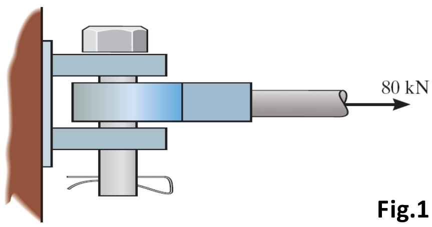


Fig.1

核心能力 2: 學會計算出結構在負載作用下產生的應力與應變量

a. 瞭解如何正確畫出結構之自由體圖(free body diagram)並列出合適的平衡方程式輔助解題

b. 學會結構在分別受到靜定與靜不定型態之軸向負載(axial load)、扭轉(torsion)與彎曲(bending)等作用時，如何計算產生之應力及應變量，並達成適當安全設計的目標

2-1. If the 20-mm diameter rod in Fig.2 is made of A-36 steel (E=200GPa), and the stiffness of the spring is k=50 MN/m. Determine the displacement of end A when the 60 kN force is applied. (計算-axial load-靜定)

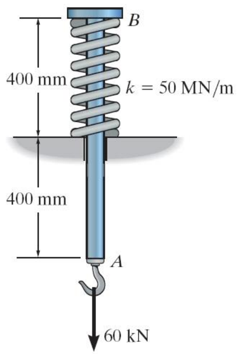


Fig.2

2-2. The A-36 steel bar shown in Fig.3 is constrained to just fit between two fixed supports when T_1=30°C. If the temperature is raised to T_2=60°C, determine the average normal thermal stress developed in the bar. Take E_{st}=200GPa, $\alpha_{st}=12(10)^{-6}$/°C. (計算-axial load-熱效應+靜不定)

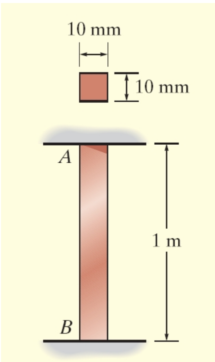


Fig.3

2-3. A solid steel shaft AB shown in Fig.4 is to be used to transmit 3750 W from the motor M to which it is attached. If the shaft rotates at ω=175 rpm and the steel has an allowable shear stress of τ_{allow}=100 MPa, determine the required diameter of the shaft to the nearest mm. (觀念+計算-torsion-動力傳遞元件設計)

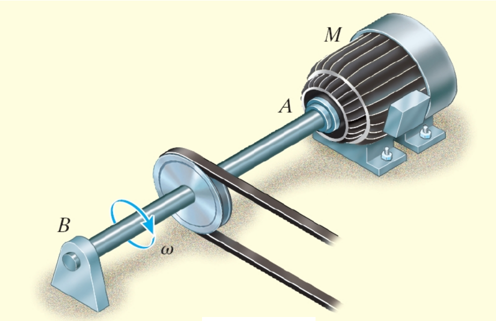


Fig.4

2-4. Draw the shear and moment diagrams for the beam shown in Fig. 5. (觀念+計算-bending-剪力彎矩圖)

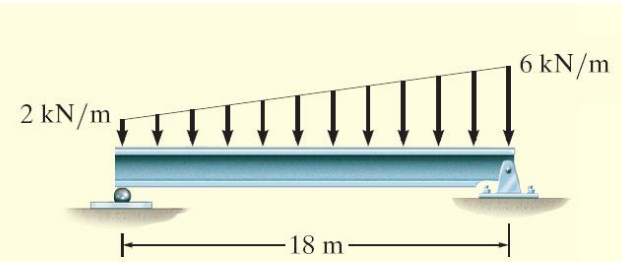
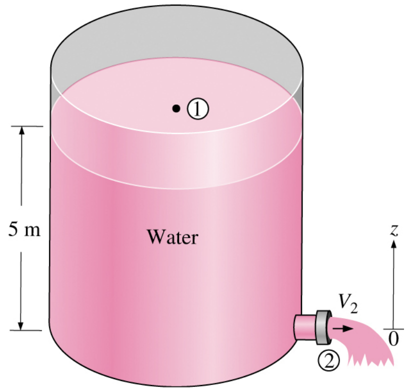


Fig.5

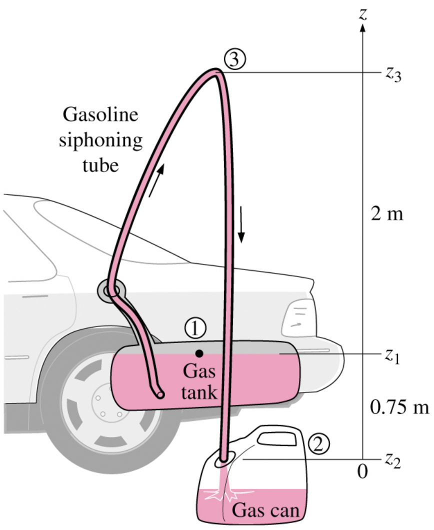
流體力學

核心能力: 藉由了解如何使用柏努力方程式來掌握流體的力與能量守恆的觀念。

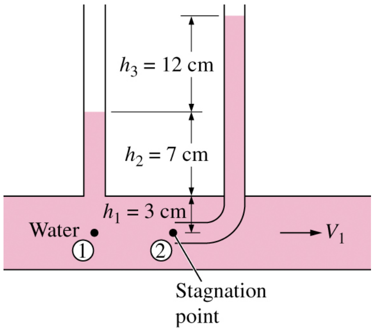
5-6(例題):一個與大氣相通的大型水槽，其中裝水，從水口算起水的高度為 5m(如圖所示)。現將接底部的出水口打開，水從平滑具導角的出口流出，試求出口的流速。



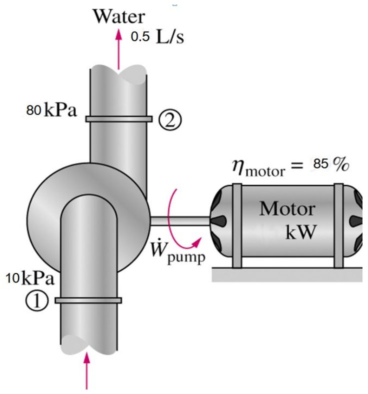
5-7(例題):一部車沒油，需要從其他人的車子裡將汽油虹吸出來(圖 5-40)。點 1(油箱內汽油的自由表面)與點 2(虹吸管的出口)之間的壓力差使的液體從較高處往較低處流動。在本題中點 2 比點 1 的高度低 0.75m，且點 3 位於點 1 以上 2m 的地方。虹吸管的直徑為 4mm，且不考慮虹吸管內的摩擦損失。試求 (a)從油箱中抽出 4L 的汽油至油罐所需最少的時間為多少? 和 (b)點 3 的壓力。P_{atm}=1atm=101.3kPa，汽油的密度為 750kg/m^3



5-8(例題):將水壓計與皮托管附接在一條水平的水管上，如圖所示，來量測淨壓與滯壓。對顯示的水柱高，試求水管中心的速度。



5-12(例題): 雨刷水管系統所使用的泵，由 54 W 的馬達帶動其效率為 85%。水經過泵的流率為 0.5 L/s，管路入口與出口的直徑相等，且經過泵的高度差忽略。如果量測泵的入口與出口絕對壓力分別為 10 kPa 和 80 kPa，試求(a)泵的機械效率；(b)當雨刷水經過泵時，由於泵的機械效能造成的溫度上升。



核心能力: 藉由掌握如何計算管路中的壓差的能力來了解管壁摩擦(Major loss)與接管段差(Minor loss)的觀念。

8-1(例題):溫度 20˚C 的油(□=888kg/m^3,□=0.800kg/m.s)，穩定流經一段直徑 5cm，長 40m 的管路。入口及出口的壓力分別為 745 及 97kPa。求流率，假設管路為(a)水平；(b)15˚朝下；(c)15˚朝上。驗證是否為層流。

8-2(例題): 溫度 40˚F 的水(□=62.42 lbm/ft^3,□=1.038X10^{-3} lbm/ft.s)，穩定流經一段直徑 0.12in，長 30ft 的水平路，平均速度 3.0ft/s。求(a)水頭損失；(b)壓力降；(c)克服壓力降所需的加壓功率。

8-3(例題): 溫度 60˚F 的水(□=62.36 lbm/ft^3,□=7.536X10^{-3} lbm/ft.s)在直徑 0.12in 的水平鋼管內流動，流率 0.2ft^3/s。求壓力降、水頭損失，以及流過 200ft 段落所需的加壓功率。

8-6(例題):一直徑 6cm 的水平水管逐漸擴張成 9cm。擴張段的管壁與水平成 30˚。擴張前的平均速度與壓力分別為 7m/s 與 150kPa。求在擴張段的水頭損失與擴張後的壓力。

動力學(Engineering Mechanics : Dynamics)

核心能力 1：質點運動力學：能夠描述質點之加速度運動，並列出運動方程式。

1. The 100-kg block *A* show in Fig.1. is released from rest. If the masses of the pulleys and the cord are neglected, determine the speed of the 20-kg block *B* in 2 s.

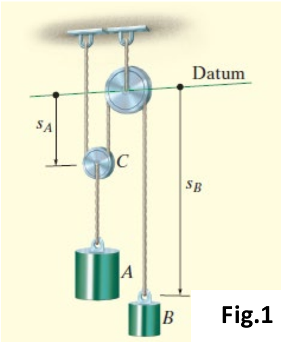


Fig.1

2. The 17.5-kN automobile show in Fig.2 is traveling down the 10° inclined road at a speed of 6 m/s. if the driver jams on the brakes, causing his wheels to lock, determine how far *s* his tires skid on the road. The coefficient of the kinetic friction between the wheels and the road is μ_k = 0.5.

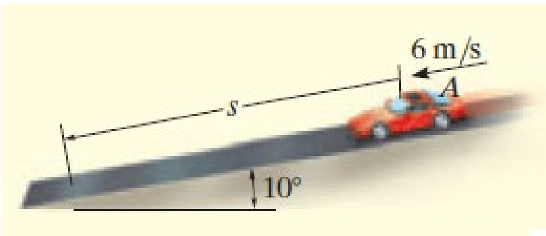


Fig.2

3. The engine of the 1750-kg car is generating a constant power of 37.5 kW while the car is traveling up the slope with a constant speed. If the engine is operating with an efficiency of ε=0.8, determine the speed of the car. Neglect drag and rolling resistance.

Fig.3

4. The 15-Mg boxcar A is coasting at 1.5 m/s on the horizontal track when it

encounters a 12-Mg tank B coasting at 0.75 m/s toward it as shown in Fig.4. If the cars meet and couple together, determine (a) the speed of both cars just after the coupling, and (b) the average force between them if the coupling takes place in 0.8 s.

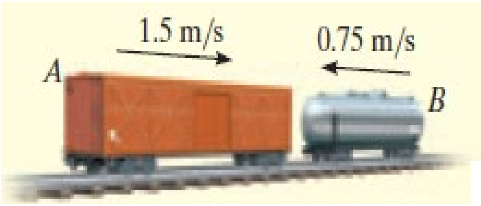


Fig.4

核心能力 2：剛體平面運動力學：能夠描述剛體之平面加速度運動，並列出運動方程式。

5. The link shown in Fig.5 is guided by two block *A* and *B*, which move in the fixed slots. If the velocity of *A* is 2 m/s downward, determine the velocity of B at the instant $\theta = 45°$.

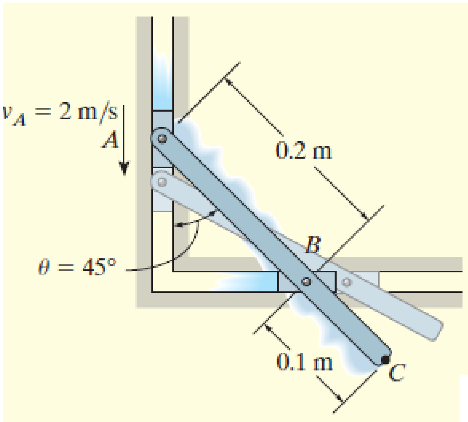


Fig.5

6. At a given instant, the cylinder or radius r, shown in Fig.6, has an angular velocity ω and angular acceleration α . Determine the velocity and acceleration of its center G and the acceleration of the center point at A if it rolls without slipping.

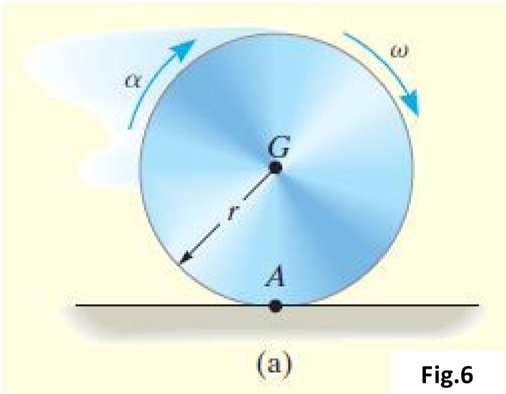


Fig.6

7. The slender rod shown in Fig. 7 has a mass *m* and length *l* and is released from rest when $\theta = 0^o$. Determine the horizontal and vertical components of force which the pin at A exerts on the rod.

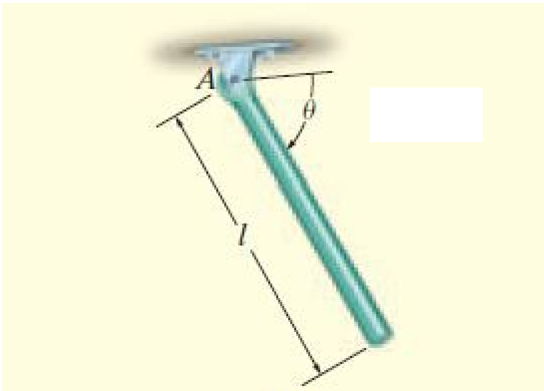


Fig.7

8. The 25-kg wheel shown in Fig.8 has a radius of gyration k_G =0.2m. If a 50 N-m couple moment is applied to the wheel ,determine the acceleration of its mass center G. The coefficients of static and kinetic friction between the wheel and the plane at A are μ_s =0.3 and μ_k =0.25, respectively.

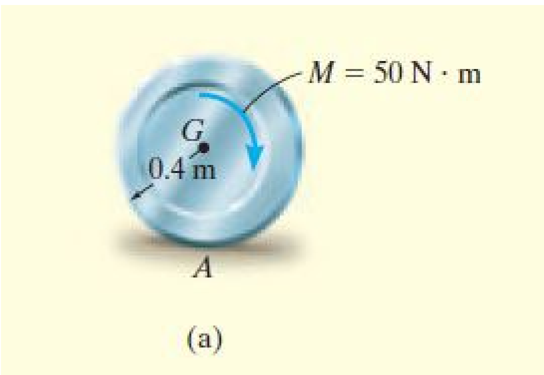


Fig.8

核心能力: 學會電晶體運算

- 電晶體的偏壓:
  - 電晶體的直流運算
  - 電路的電流及電壓運算
- 電晶體的小訊號放大:
  - 電晶體的交流等效電路
  - 電路的交流動號放大器設計及運算

1.

Select a minimum value for the emitter bypass capacitor, C_2, in Figure 6–16 if the amplifier must operate over a frequency range from 200 Hz to 10 kHz.

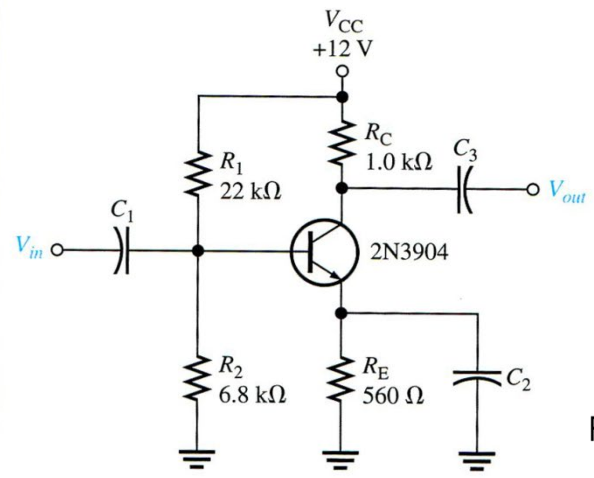
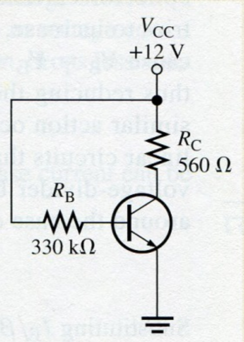


Figure 6-16

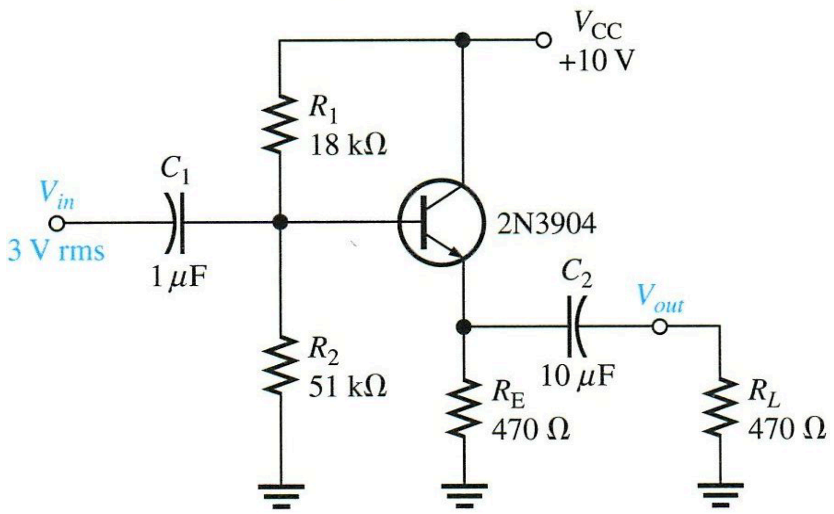
2.

Determine how much the Q-point (I_C, V_{CE}) for the circuit in Figure 5–21 will change over a temperature range where β_{DC} increases from 100 to 200.

FIGURE 5–21

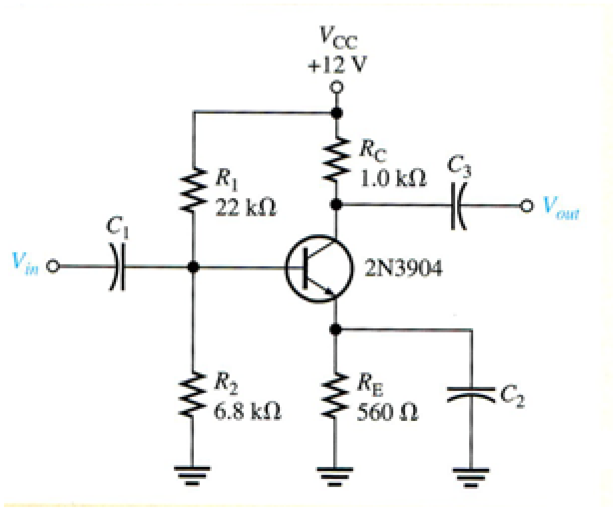


3. Determine the total input resistance of the emitter-follower in figure. Also find the voltage gain, current gain, and power gain in terms of power delivered to the load, R_L. Assume $\beta_{ac} = 175$ and that the capacitive reactances are negligible at the frequency of operation.



4 Determine the voltage gain of the following amplifer. It is known that

$\beta_{DC} = \beta_{ac} = 100$, $V_{CE(sat)} = 0.2$V. The capacitive reactances are negligible at the frequency of operation.



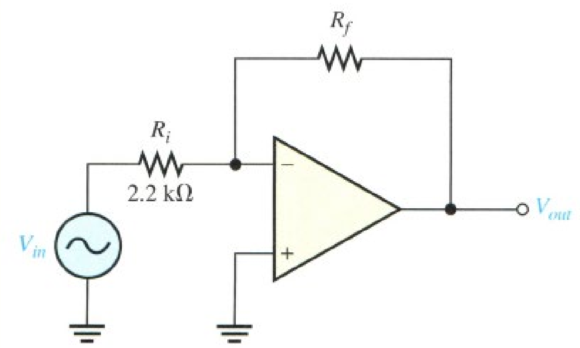
核心能力: 學會運算放大器運算

- 運算放大器的基本特性:
  - 輸入電阻無限大
  - 開迴路增益無限大
  - …
- 運算放大器的應用::
  - 反相及非反相放大器
  - 微分器、積分器及其他應用

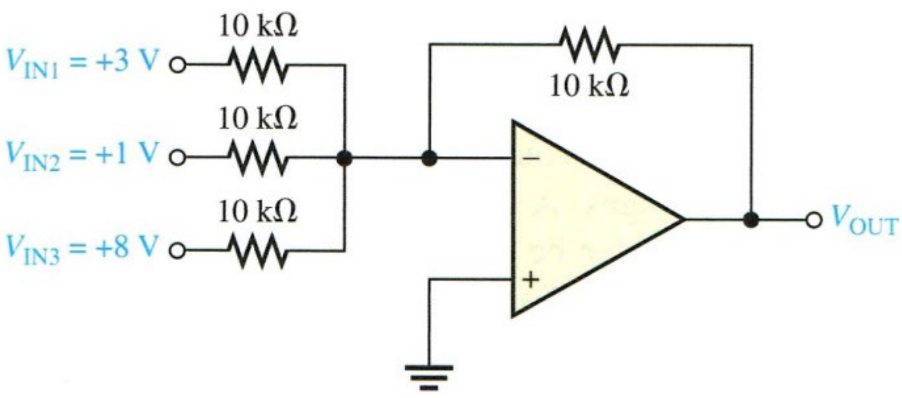
1.

Given the op-amp configuration in Figure 12–21, determine the value of R_f required to produce a closed-loop voltage gain of -100.

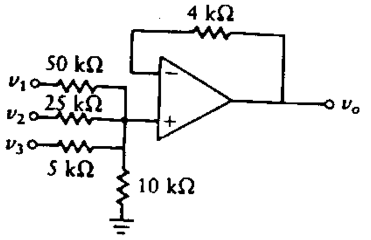
FIGURE 12–21



2. Determine the output voltage of the following circuit.

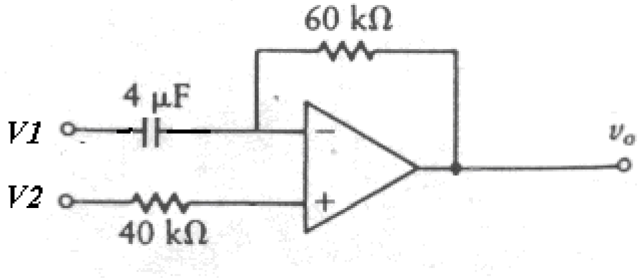


3. 試算出 v_o 值為何？



4. (Hint: $Z_C = \frac{1}{CS}$) , Determine $v_o(t)$ 35%

(不能套公式，用推導的)

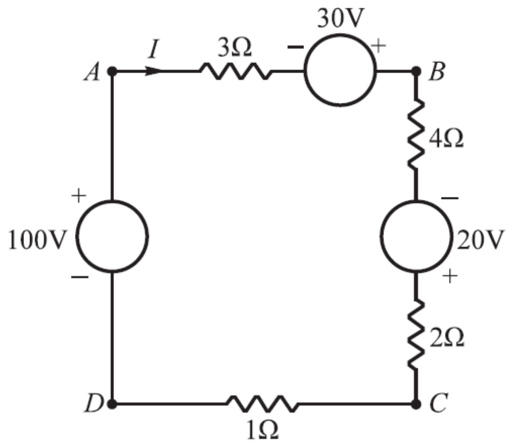


I. 核心能力: 學會電路基礎分析計算方法

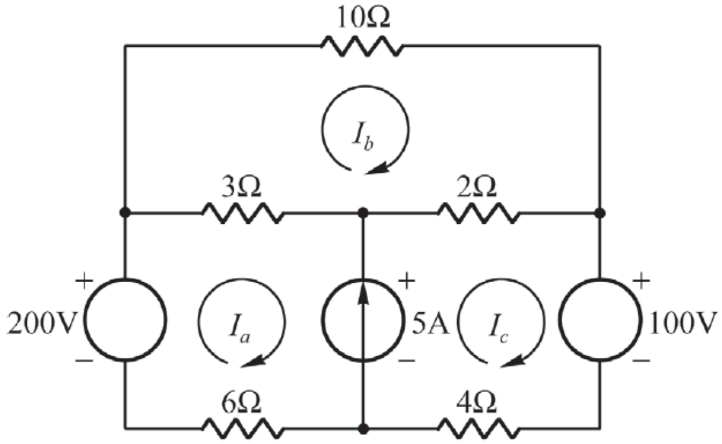
– 克希荷夫(Kirchhoff)電壓、電流定理
– 節點(node)分析法、網目(mesh)分析法
– 戴維寧(Thevenin)、諾頓(Norton)等效電路定理

• 計算題:

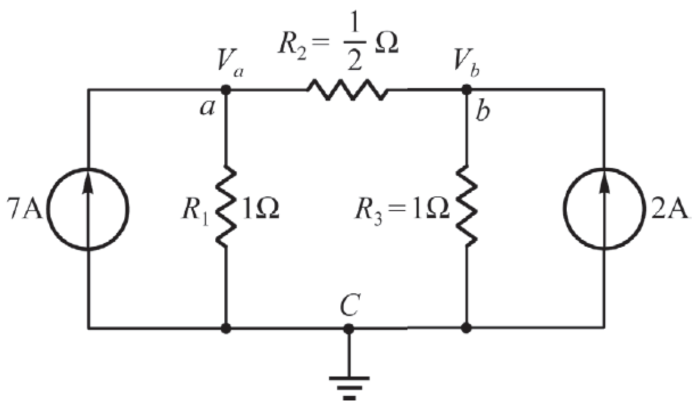
1. 試以 Kirchoff's voltage law (KVL)求下圖中電流 I 值?



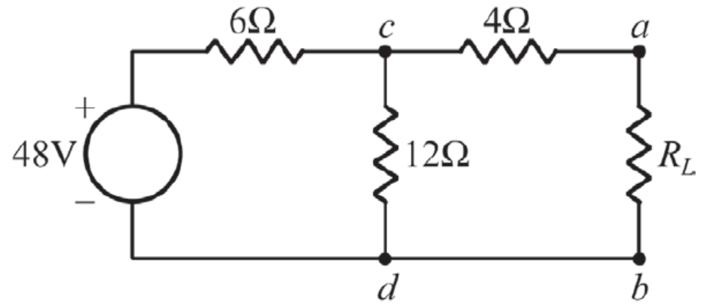
2. 試求下圖中之網目電流 I_a、I_b、I_c 值？



3. 使用節點電壓法(node voltage method)求下圖中節點電壓 V_a、V_b 值？



4. 使用戴維寧定理(Thevenin's theorem)，求下圖電路 R_L=8Ω 時之流經 R_L 之電流值？

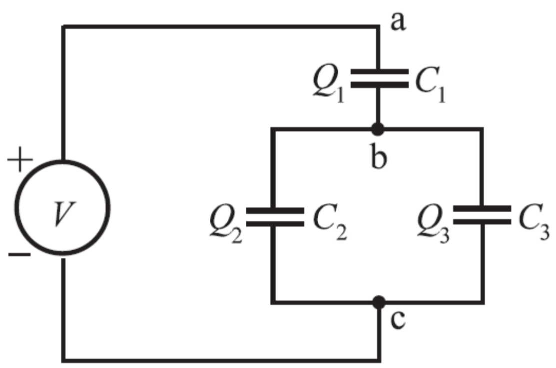


II. 核心能力: 認識電路儲能元件(電容&電感)之特性

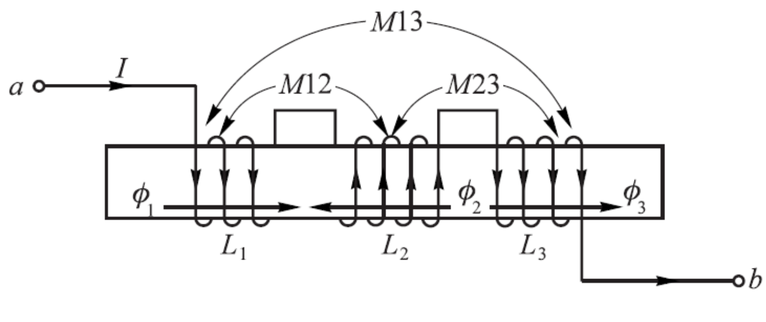
- 元件串聯、並聯之效應
- 元件電壓與電流之關係
- 暫態與穩態響應分析

- 計算題:

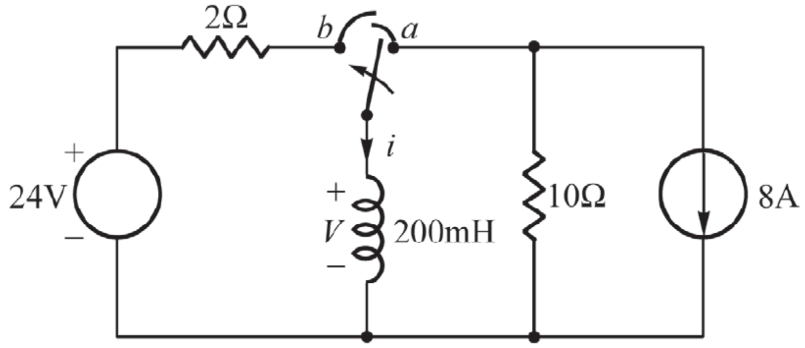
1. 電路圖中 $C_1=C_2=C_3=1\mu F$，接上電源後 C_1 電容兩端之電壓達 100V，試求(1)a、c 端點間之總電容量值(2) C_1 電容之儲存電荷量 Q_1 值(3) C_2 電容之儲存電荷量 Q_2 值(3) C_2 與 C_3=並聯部分兩端之電壓 Vbc 值(4)電源電壓 V 值



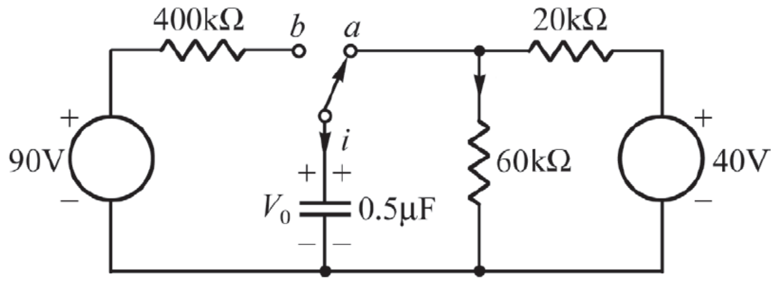
2. 如下圖所示，有三個電感串聯其電感量分別為 L_1=5H，L_2=10H，L_3=8H，其間之互感量(mutual inductance)分別為 M_{12}=3H，M_{23}=4H，M_{13}=2H，試求串聯後 a、b 間之等值電感量值？



3. 如下圖電路中，若開關先已置於 a 點很久，當在 t=0 瞬間開關扳至 b 位置，試求 $t > 0$ 時(1)電感電流 $i(t)$ 方程式?(2)開關扳至 b 位置以後電感兩端的初始電壓值?



4. 如下圖電路中，若開關先已置於 a 點很久，當在 t=0 瞬間開關扳至 b 位置，試求 $t > 0$ 時(1)電容電壓 $v(t)$ 方程式?(2)電容電流 $i(t)$方程式?



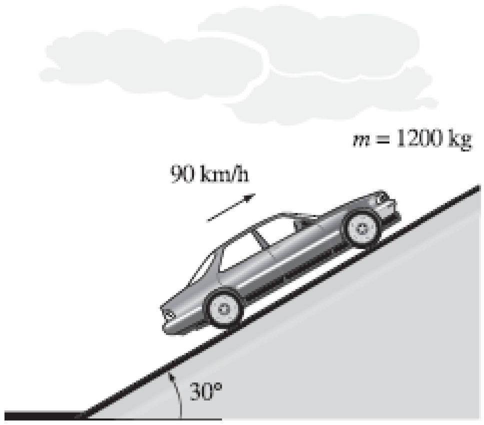
熱力學

I. 核心能力: 能量守恆

- 能量單位計算(單位質量能量、總能、動能、位能、功及功率)
- 能量守恆
- 計算效率

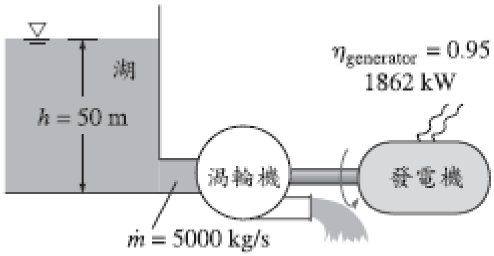
1. 一個穩定風速8.5 m/s 的場址欲評估是否可設為風力農場。求出下列的風能：(a) 每單位質量；(b) 10 kg 的質量；(c)空氣流率為1154 kg/s。

2. 一部1200 kg 的車子在水平路面，在20 s 內由靜止加速到90 km/h 所需的功率。若車子以90 km/h 的速度穩定前進，現在車子開始爬上30º 的斜坡，如果車子在爬坡過程的速度維持不變，試求引擎需要多少功率？若車子爬坡上升的垂直高度為20公尺，試求引擎需要做多少功。



3. 一個剛性容器中裝有熱流體，並使用葉輪攪拌冷卻與電熱器加熱。一開始，流體的內能為800 kJ，在冷卻過程中，流體散失500 kJ 的熱量，而葉輪對流體作功100 kJ。電熱器的電流為2A，電壓120V，加熱時間為10分鐘。若葉輪儲存的能量忽略不計，試求流體最終的內能。

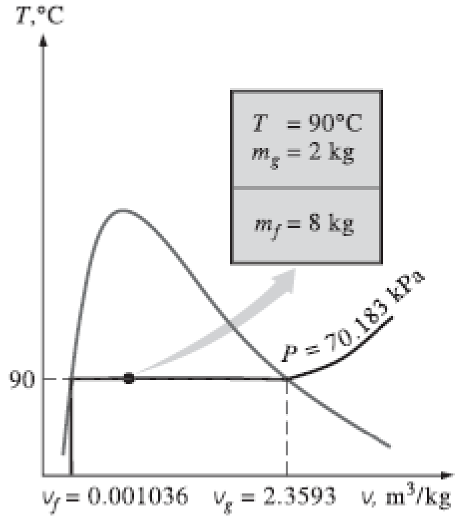
4. 在水深50 m 處的水力渦輪機－發電機利用湖中的水來發電（如圖2-60 所示）。水的供給率為5000 kg/s。若發電機的效率為95%，量測到的發電量為1862 kW，試求：(a) 渦輪機－發電機的總效率；(b) 渦輪機的機械效率；(c) 渦輪機傳送至發電機的軸功。



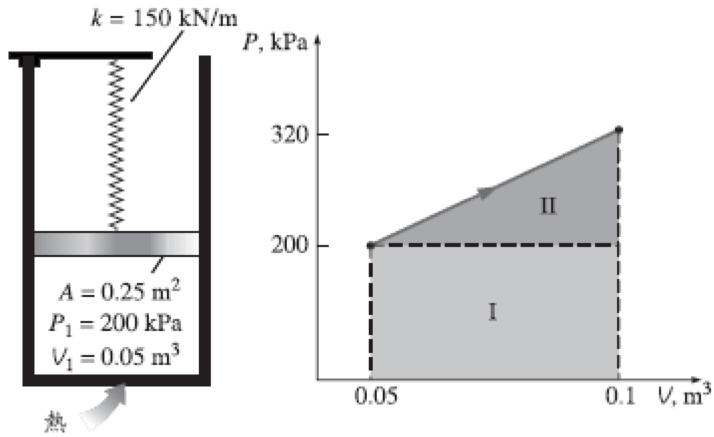
II. 核心能力：封閉系統與開放系統能量分析

- 狀態表熱力性質的使用
- 封閉系統的能量分析
- 開放系統能量分析

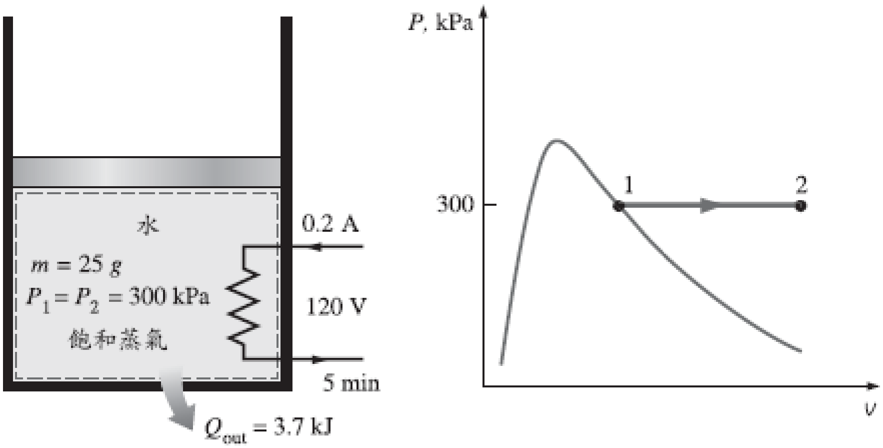
1. 一剛槽內含10 kg、90°C 的水。若8 kg 的水為液態，而其餘為蒸氣，試求：(a) 剛槽內的壓力；(b) 平均比容(c) 剛槽的體積(d)單位質量內能(e)總內能(f) 單位質量的焓(g) 總焓。



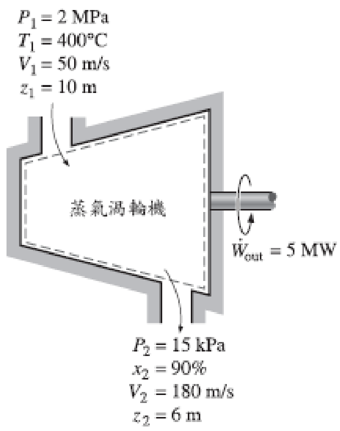
2. 如圖4-10 所示，一組活塞－汽缸裝置內含200 kPa、0.05 m3 的氣體，此時，一個線性彈簧（彈簧常數為150 kN/m）置於活塞上方，但與活塞之間沒有接觸力。此時，熱從外界傳入汽缸，使得汽缸內氣體膨脹，並壓縮彈簧，直至汽缸內體積增加為原來的兩倍。假設汽缸的截面積為0.25 m2，試求：(a) 最終狀態的汽缸壓力；(b) 系統對外界所作的功；(c) 被彈簧吸收的功。



3. 如圖4-13 所示，一個活塞－汽缸裝置內含25 g 的飽和水蒸氣（300 kPa）。汽缸內以電阻絲加熱氣體5 分鐘，電阻絲的電流為0.2 A，電壓為120 V，在此時間內，汽缸對外的熱散失為3.7kJ。(a) 定壓狀態下，證明此封閉系統的邊界功W_b、內能變化ΔU，可以ΔH表示。(b) 求出最終狀態下水蒸氣的溫度。



4. 一絕熱的蒸氣渦輪機之功率輸出為5 MW，而水蒸氣的入口與出口情況如圖5-28 所示。(a) 比較Δh、Δke 及Δpe 的大小；(b) 求出每單位質量水蒸氣流經渦輪機所作的功；(c) 計算水蒸氣的質量流率。



靜力學

核心能力 1: 計算施予物件的合力(ΣF)與合力矩(ΣM)，轉換為等效合力與等效合力矩的表現。

- **觀念題**

1. As shown in **Figure 1-1**, determine the resultant moment of the four forces acting on the rod about point O.
2. Determine the moment produced by the force **F** in **Figure 1-2** about point O. Express the result as a Cartesian vector.

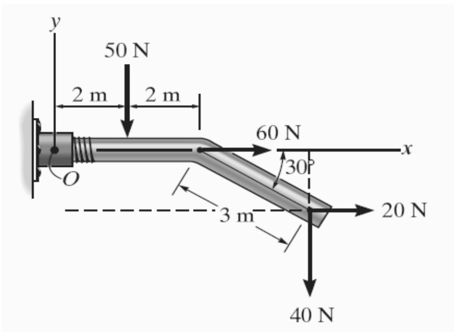


Figure 1-1

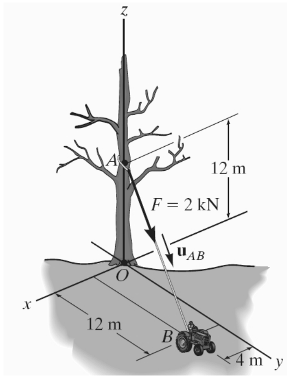


Figure 1-2

3. Replace the force and couple moment system acting on the beam in **Figure 1-3** by an equivalent resultant force, and find where its line of action intersects the beam, measure from point O.

4. Replace the force and couple system acting on the member in **Figure 1-4** by an equivalent resultant force and couple moment acting at point O.

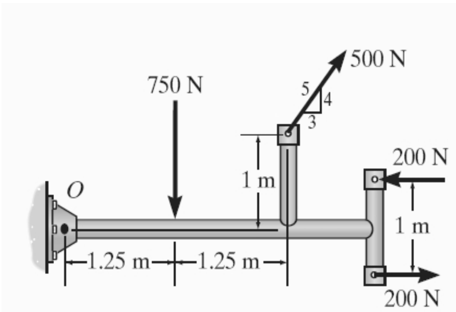


Figure 1-3

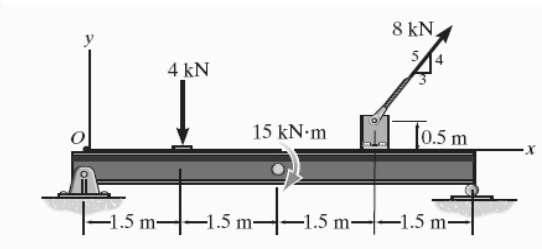


Figure 1-4

核心能力 **2:** 能夠建立物件的自由體圖，進行力(ΣF=0)與力矩(ΣM=0)的平衡。

- **觀念題**

1. Determine the horizontal and vertical components of reaction for the beam loaded as shown in **Figure 2-1**. Neglect the weight of the beam in the calculations.
2. As shown in **Figure 2-2.** The lever ABC is pin-supported at A and connected to a short link BD. If the weight of the members is negligible, determine the force of the pin on the lever at A.

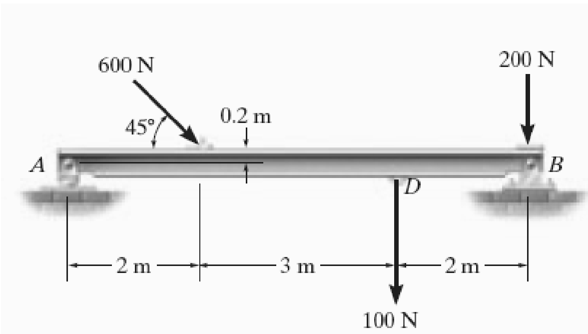


Figure 2-1

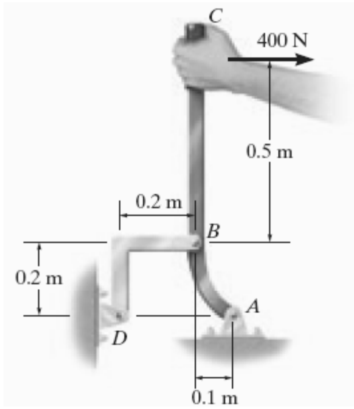


Figure 2-2

3. The uniform crate has a mass of 20 kg. If a force P = 80 N is applied on to the crate, determine if it remains in equilibrium as shown in **Figure 2-3.** The coefficient of static friction is μ = 0.3.
4. Determine the force in each member of the truss and indicate whether the members are in tension or compression as shown in **Figure 2-4**.

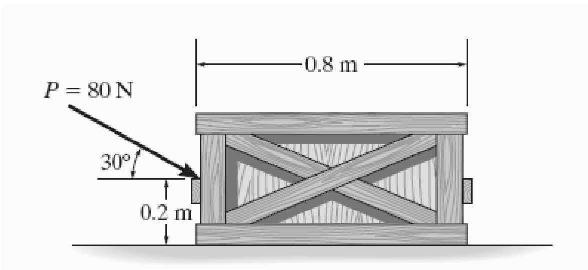


Figure 2-3

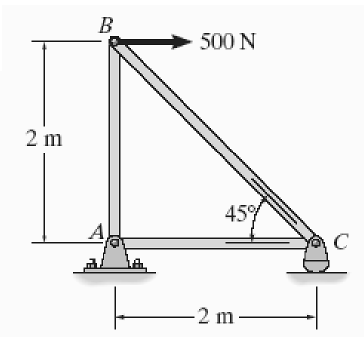


Figure 2-4